

Cheat Sheet for Fractions

Converting Fractions:

1. Mixed to Improper:
 - a. Multiply the whole number by the denominator (bottom number)
 - b. Add the numerator (top number)
 - c. This number becomes the new numerator
 - d. The denominator stays the same

Example:

$$3\frac{2}{5} \longrightarrow \begin{array}{l} 3 \times 5 = 15 \\ 15 + 2 = 17 \end{array} \longrightarrow \frac{17}{5}$$

2. Improper to Mixed:
 - a. Divide the numerator by the denominator
 - b. The number of full times the denominator fits into the numerator is your whole number
 - c. The remainder goes into a fraction over the original denominator

Example:

$$\frac{17}{5} \longrightarrow \begin{array}{r} 3 \\ 5 \overline{)17} \\ \underline{-15} \\ 2 \end{array} \longrightarrow 3\frac{2}{5}$$

Prime Numbers:

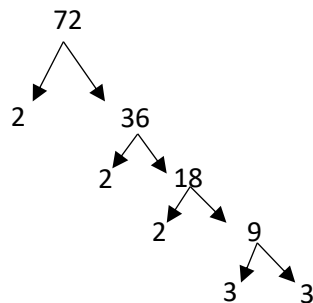
- Prime numbers are only dividable by itself and 1

Example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Prime Factorization:

- Tree!
- Start by dividing the number by the first prime number (2)
- When you can't divide by a prime number, you move on to the next prime number possible

Example:



- Write as each prime number multiplies by the next
- Simplify with exponents

Example: $2 \times 2 \times 2 \times 3 \times 3$

$$2^3 \times 3^2$$

Writing Fractions in Lowest Terms

1. Divide the numerator (top number) and denominator (bottom number) by common factors

Example:

$$\frac{24}{36} = \frac{12}{18} = \frac{4}{6} = \frac{2}{3}$$

2. Find lowest terms using prime factorization
 - a. Do a prime factorization tree for both numbers in the fraction
 - b. Write the product of each tree into a fraction
 - c. Divide the terms

Example: $\frac{24}{36}$

$$\frac{24}{36} = \frac{2 \times 2 \times 2 \times \cancel{3}}{2 \times 2 \times \cancel{3} \times 3} = \frac{2}{3}$$

Multiplying Fractions

1. Proper Fractions:
 - a. Multiply the numerators
 - b. Multiply the denominators
 - c. Write in lowest terms

Example: $\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$ ← lowest terms

2. Improper Fractions:
 - a. Multiply the numerators
 - b. Multiply the denominators
 - c. Use long division to write as a mixed number
 - d. Write fraction in lowest terms

Example: $\frac{6}{4} \times \frac{5}{2} = \frac{30}{8} \longrightarrow 8 \overline{) \begin{array}{r} 30 \\ -24 \\ \hline 6 \end{array}} \longrightarrow 3 \frac{6}{8} \xrightarrow{\div 2} 3 \frac{3}{4}$

3. Mixed Fractions:

- Write each mixed number as an improper fraction
- Multiply the numerators
- Multiply the denominators
- Use long division to write as a mixed number
- Write fraction in lowest terms

Example: $2 \frac{1}{5} \times 2 \frac{2}{3} = \frac{11}{5} \times \frac{8}{3} = \frac{88}{15} \longrightarrow 15 \overline{) \begin{array}{r} 88 \\ -75 \\ \hline 13 \end{array}} \longrightarrow 5 \frac{13}{15} \xrightarrow{\div 5} 5 \frac{1}{3}$

Dividing Fractions

1. Dividing Proper or Improper Fractions:

- Change the division sign to multiplication
- Leave the first term the same
- Find the reciprocal of the second term (flip)

Example: $\frac{3}{5} \div \frac{1}{3} = \frac{3}{5} \times \frac{3}{1} = \frac{9}{5}$

- Simplify by writing the fraction in lowest terms or by writing as a mixed number

Example: $\frac{9}{5} \longrightarrow 5 \overline{) \begin{array}{r} 9 \\ -5 \\ \hline 4 \end{array}} \longrightarrow 1 \frac{4}{5}$

2. Dividing Mixed Numbers:

- Change the mixed number to an improper fraction
- Change the division sign to multiplication
- Leave the first term the same
- Find the reciprocal of the second term (flip)
- Simplify by writing the fraction in lowest terms or by writing as a mixed number

Example: $3 \frac{5}{9} \div 2 \frac{2}{5} = \frac{32}{9} \div \frac{12}{5} = \frac{32}{9} \times \frac{5}{12} = \frac{160}{108} \xrightarrow{\div 2} \frac{80}{54} \xrightarrow{\div 2} \frac{40}{27} \longrightarrow 27 \overline{) \begin{array}{r} 140 \\ -27 \\ \hline 13 \end{array}} \longrightarrow 1 \frac{13}{27}$

Adding/Subtracting Fractions:

1. Adding/Subtracting Like Fractions:

- Add or subtract the numerators
- Leave the bottom numbers the same

c. Simplify by writing in lowest terms or as a mixed number

Example: $\frac{2}{8} + \frac{4}{8} = \frac{6}{8} = \frac{3}{4}$

$\div 2$

$$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

2. Finding the Lowest Common Multiple:

a. List Method

- i. List the first few multiples of each denominator
- ii. Find the lowest number they have in common

Example: $\frac{1}{4}, \frac{2}{6}$

4, 8, 12, 16, 20, 24, 28, 32
6, 12, 18, 24, 30

b. Dividing Prime Numbers Method

- i. Start by trying to divide by the first prime number
- ii. Continue dividing by prime numbers until all quotients are 1
- iii. Multiply all prime numbers used to get lowest common multiple

Example: 9, 15

3	9	15
3	3	5
5	1	5
	1	1

$3 \times 3 \times 5 = 45$ is the lowest common multiple

3. Adding/Subtracting Unlike Fractions:

- a. Find the lowest common multiple
- b. Rewrite the fractions with the lowest common multiple as the denominator
- c. Add/subtract the numerators (top numbers)
- d. Simplify by writing in lowest terms or by writing as a mixed number

Example:

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{10}$$

2	3	4	10
2	3	2	5
3	3	1	5
5	1	1	5
	1	1	1

$2 \times 2 \times 3 \times 5 = 60$ is the lowest common multiple

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{10} \xrightarrow{\begin{matrix} \times 20 \\ \times 6 \\ \times 15 \end{matrix}} \frac{20}{60} + \frac{15}{60} + \frac{6}{60} \longrightarrow = \frac{41}{60} \longleftarrow \text{lowest terms}$$

4. Adding/Subtracting Mixed Numbers:

- Change the Mixed Number to an improper fraction
- Find the lowest common multiple of the denominators (bottom number)
- Requite the fractions with the lowest common multiple as the denominators
- Add or subtract the numerators (top number)
- Simplify by writing in lowest terms or by writing as a mixed number

Example:

$$8\frac{5}{8} - 3\frac{11}{12} = \frac{69}{8} - \frac{47}{12} \longrightarrow \text{LCM} = 8, 16, \textcircled{24}, 32, 40, 48 \longrightarrow \frac{69}{8} \xrightarrow{\times 3} \frac{207}{24} - \frac{47}{12} \xrightarrow{\times 2} \frac{94}{24} = \frac{113}{24} \xrightarrow{\text{improper}}$$

$$\begin{array}{r} 4 \\ 24 \overline{) 113} \\ \underline{-96} \\ 17 \end{array} \longrightarrow 4\frac{17}{24}$$

Estimating Fraction Equations:

- Round the mixed numbers/fractions to a whole number
- Estimate the answer
- Use estimate to check if your exact answer is reasonable
 - If the numerator of the fraction is at least half of the denominator, you round the whole number up
 - If the numerator of the fraction is less than half of the denominator, you round down (leave the whole number as is)

Example:

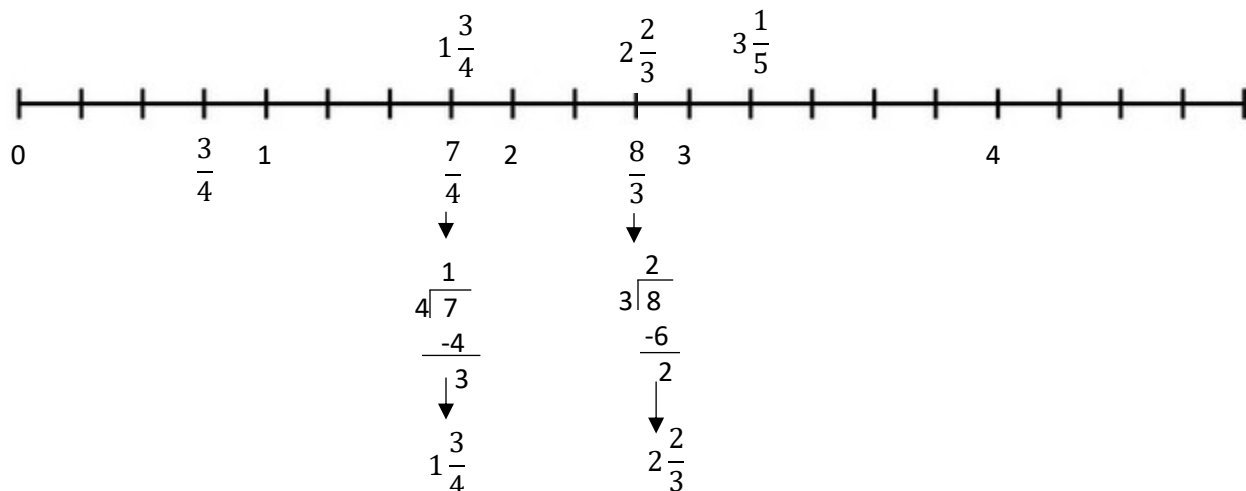
$$8\frac{1}{3} - 2\frac{5}{6} = 8 - 3 = 5$$

less more

$$5\frac{5}{12} + 2\frac{5}{8} = 5 + 3 = 8$$

less more

Locating Fractions on a Number Line:



Greater than, Less than or Equal:

Greater than >

Less than <

Equal to =

Alligator wants to eat the bigger one

1. Find lowest common multiple
2. Rewrite fractions with lowest common multiple as the denominator
3. Compare fractions

Example: $\frac{2}{3} < \frac{3}{4}$

LCM: 3, 6, 9, 12, 15, 21, 24
4, 8, 12, 16, 20, 24

$$\frac{2}{3} < \frac{3}{4} = \frac{8}{12} < \frac{9}{12}$$

Diagram illustrating the comparison of fractions $\frac{2}{3}$ and $\frac{3}{4}$ by finding a common denominator (LCM = 12).

For $\frac{2}{3}$, multiply numerator and denominator by 4: $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$.

For $\frac{3}{4}$, multiply numerator and denominator by 3: $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$.

Since $\frac{8}{12} < \frac{9}{12}$, the fraction $\frac{3}{4}$ is greater than $\frac{2}{3}$. The final comparison $\frac{8}{12} < \frac{9}{12}$ is circled, with an arrow pointing to it labeled "Greater than".